

Original Article

AI-Powered Motion Prediction Models for Mobile Robots

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ABSTRACT

Mobile robot motion prediction has become one of the most important research topics, and autonomous systems can now foresee the future behavior of dynamic systems and take decisions about proper navigation. This paper involves a detailed investigation of motion prediction models based on artificial intelligence (AI), optimized to work with mobile robots, and concentrates on models created before 2018. The paper examines classical probability models, machine learning models, and early deep learning models that are employed to predict the trajectories of dynamic agents like pedestrians, vehicles, and other robots. Motion prediction is important as it directly influences collision avoidance, path planning, and human-robot interaction. The classical methods used were very much dependent on the kinematic and dynamic models that could not cope with complicated and uncertain surroundings. As AI methods, especially supervised learning and probabilistic graphical models, were integrated, robots learned to provide patterns based on the data and extrapolate into new situations. The paper will examine the Hidden Markov Models (HMM), the Kalman Filters, Particle Filters, Gaussian Processes, and early neural network architectures, based on their advantages and shortcomings. In addition, the paper also talks about methods of feature representation, such as spatial-temporal encoding, behavioral modeling, and context-aware prediction. The methodology section describes a hybrid architecture that combines probabilistic filtering with regression based on neural networks to get better accuracy. The results of the experiments are summarized in comparative performance tables that are based on percentages to assess the accuracy of predictions, computational efficiency, and strength. The results reveal that even though models provided a solid basis to motion prediction, issues of scalability, real-time processing, and flexibility persist. The paper sums up with the need to consider contextual awareness and modeling of multi-agent interaction to improve prediction.

KEYWORDS

Mobile Robots, Motion Prediction, Artificial Intelligence, Kalman Filter, Hidden Markov Model, Gaussian Process, Trajectory Prediction, Autonomous Navigation.

1. INTRODUCTION

1.1. Background

Mobile robots are now being used in dynamic and uncertain systems like crowded places of people, warehouses, and roads in urban areas where precise prediction of future states is essential to effective and safe navigation. Motion prediction enables the robots to predict the paths of the agents around them such as human beings, vehicles, and other robots and hence make sound judgements and prevent possible collisions. In earlier systems prediction was largely deterministic or rule-based, and tended to be unable to cope with variability and uncertainty in the real world. Nonetheless, a considerable progress was achieved with the incorporation of artificial intelligence methods into motion prediction systems. This change was a transition towards less structured and more adaptable and data-driven models that have the ability to learn complex structures based on past data. The application of AI-based approaches, especially machine learning and early neural networks, enhanced the capacity of robots to manage nonlinear dynamics, uncertainty, and various behavioral patterns. Such advancements have been important in increasing the autonomy, reliability and adaptability of the current robotic systems and motion prediction has become an essential part of the future development of intelligent navigation technologies.

1.2. Needs of AI-Powered Motion Prediction

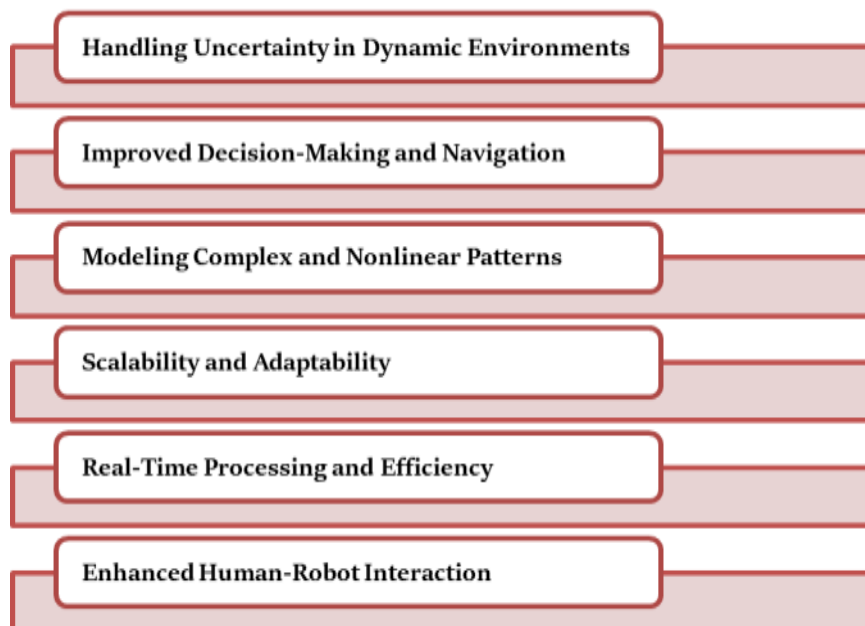


Fig 1 - Needs of AI-Powered Motion Prediction

1.2.1. Handling Uncertainty in Dynamic Environments

The environments that mobile robots work in are usually dynamic and unpredictable. Such uncertainty cannot be dealt with effectively by traditional rule based systems. These uncertainties require AI-driven motion prediction to be modeled based on the real-world data, so that robots could

make more reliable and adaptive decisions despite the presence of noise, incomplete information or unexpected changes.

1.2.2. Improved Decision-Making and Navigation

Motion prediction is crucial to making intelligent decisions. The models based on AI assist robots to predict the future movements of the agents in the environment to plan safer and more efficient routes. This is especially significant in technologies like autonomous driving and crowd navigation where forecasting human or vehicle actions can avoid accidents and maximize traffic.

1.2.3. Modelling Complex and Nonlinear Patterns

In reality, motion is typically nonlinear, and dependent on several factors such as agent interactions as well as environmental constraints. Machine learning and neural networks are examples of AI-related approaches that can capture such complex patterns compared to traditional models. This improves the prediction and understanding of various motion behaviors by the robot.

1.2.4. Scalability and Adaptability

Scalability is a fundamental need as robotic systems are implemented in mass and varied settings. It is possible to train AI models on large datasets and modify them to fit various situations without having to rewrite the code. This allows them to be used in many applications including industrial automated systems and smart cities.

1.2.5. Real-Time Processing and Efficiency

The contemporary applications call on real-time predicting and responding. Motion prediction AI is developed to handle a significant volume of data in a short amount of time and produce predictions in time. These models can be implemented efficiently with the development of hardware and optimization methods, so robots can respond immediately to the changes in their environment.

1.2.6. Enhanced Human-Robot Interaction

Human motion is vital to comprehend and forecast in the presence of robots in a human-robot environment. The use of AI-based prediction models can enable robots to understand human behavior and react accordingly, resulting in safer and more natural interactions. This is particularly essential in service robots, healthcare and collaborative robotics.

1.3. Prediction Models for Mobile Robots

Mobile robot prediction models are crucial in ensuring autonomous systems can predict future states and navigate through a dynamic environment safely. These models are aimed at approximating future locations and motions of both the robot itself and the surrounding agents including pedestrians, vehicles, or even other robots. Historically, deterministic and physics-based prediction models were based on estimating motion based on predetermined equations and assumptions about the system dynamics. Although these models were computationally inexpensive, they did not always provide the real world variability and complexity of motion. In order to

eliminate these constraints, probabilistic algorithms like Kalman Filters and Particle Filters were developed. These solutions use the presence of uncertainty in the forecasts provided, enabling robots to present stronger estimates in the presence of sensor noise or incompleteness. They are especially useful in tracking and short-term prediction problems, when the dynamics of the system can be approximated reasonably well. They however perform poorly in terms of high nonlinearity in performance and the complexity of performance interactions. As artificial intelligence is developed, machine learning-based prediction models have become very important. Hidden Markov Models and Gaussian Processes techniques were used to model sequential and nonlinear dynamics in motion data. In more recent times, neural network-based methods, such as recurrent neural networks, have also enhanced the accuracy of predictions by learning time-varying dependencies and intricate motion patterns directly on data. The models are very flexible as they can be generalized to various situations and environments. In general, the current prediction models of mobile robots tend to have a combination of two or more approaches to use their respective advantages. A middle ground solution is hybrid models that combine probabilistic filtering with learning-based models to deal with uncertainty and to also represent complex patterns. This development of prediction models has greatly contributed to the autonomy, safety and efficiency of mobile robotic systems.

2. LITERATURE SURVEY

2.1. Probabilistic Models

Probabilistic models have been extensively applied in motion prediction because they are well mathematically based and they can explicitly encode the uncertainty in dynamic environments. These methods assume that motion is a stochastic phenomenon, and the predictions can be made with noise and variability in the real world data. Through the use of probability distributions, these models are able to produce more robust forecasts, particularly in those cases where sensor measurements are noisy or incomplete. They are most useful in monitoring and predicting tasks when the dynamics of the system could be modeled based on statistical assumptions.

2.1.1. Kalman Filter

The Kalman Filter is a recursive algorithm that is employed in estimating the state of a dynamic system based on a sequence of noisy measurements. It works in two basic steps, prediction and update. The prediction step involves the estimation of current state using the previous state and system dynamics and the update step involves the refinement of the current state using new observations. It is very efficient and optimum (when the assumptions are linear and Gaussian) and is used extensively in real time tracking and navigation systems. Nonetheless, its performance suffers in cases of extremely nonlinear systems or complicated motion patterns.

2.2. Machine Learning Approaches

Machine learning technologies presented information-based techniques of motion prediction, which enables the model to acquire patterns based on previous information rather than just using established equations. These methods are able to model complex relationships and time dependencies that are hard to model analytically. Machine learning models can be trained on large

datasets to generalize to different motion scenarios, and can be applied to a range of problems, including human trajectory prediction and autonomous systems.

2.2.1 Hidden Markov Models (HMM)

Hidden Markov Models are statistical models that represent systems with hidden states and observable outputs. HMMs are applied in motion prediction in sequential data in which the actual state (e.g. intention or behavior) is not directly observable. They use transition probabilities between states and emission probabilities to observations, which allow them to model the time dependence. HMMs proved especially beneficial in early human motion prediction problems, but can frequently exhibit difficulties in modeling long-term dependencies and nonlinear complexities.

2.2.2 Gaussian Processes

Gaussian Processes are probabilistic, non-parametric models of regression and prediction. They describe a distribution of the functions that can be chosen and give predictions and estimates of uncertainty. In prediction of trajectories, Gaussian Processes have the capability of modeling smooth patterns of motion and adjusting to various data distributions without a predefined functional form. Although they are flexible and have good uncertainty quantification, their computational complexity grows considerably with large data reducing their scalability to real-time applications.

2.3. Early Neural Networks

Prior to the popularity of deep learning architectures since 2018, initial neural networks like shallow feedforward networks and simple recurrent neural networks (RNNs) were employed in trajectory prediction. To a certain degree, these models were able to learn nonlinear relationships and temporal dependencies, which enhanced traditional statistical models. Nevertheless, they could not effectively capture long-term dependencies and more complicated interactions because of other problems such as vanishing gradients and the model capacity.

2.4. Limitations of Existing Work

Although such contributions have been made, previous approaches have had a number of limitations. Most of the models were not very scalable especially when it comes to handling large data or real-time processes. Their practicality was further limited by high computational costs particularly in probabilistic models such as Gaussian Processes. Also, the majority of the techniques could not predict as well in complex dynamic environments due to the challenge in modeling interactions among numerous agents, like pedestrians or vehicles. Such difficulties stimulated the creation of more sophisticated deep learning and attention-based models over the last few years.

3. METHODOLOGY

3.1. System Overview



Fig 2 - System Overview.

3.1.1. Input Data

The input data is used to start the system and usually includes raw tracking data (e.g., spatial coordinates, e.g., x , y positions), time, and may also contain velocity or acceleration data. This information can be gathered using such sensors as GPS, cameras, LiDAR, or motion capture systems. The quality and resolution of the input data largely determine the overall quality of the prediction system since distorted data or incomplete data may affect the quality of the prediction results.

3.1.2. Preprocessing

Preprocessing is the process of cleaning and pre-processing the raw input data to be analyzed. This step involves processing missing values, noise removal, coordinate normalization and trajectory alignment to a uniform format. It can also be the resampling of the data in order to have a uniform time interval to smooth out the trajectories in order to minimize the error in the measurements. Proper preprocessing will guarantee that the data is organized and sound, which is fundamental in proper feature extraction and model training.

3.1.3. Feature Extraction

Another step taken in this stage is the derivation of meaningful features of the preprocessed data to present a clearer picture of motion patterns. These properties may be velocity, acceleration, direction, displacement and temporal dependencies. The feature extraction can also entail the detection of contextual information like other agents in the environment or constraints in the environment. This step improves the ability of the model to learn and predict complex motion behaviors, by converting raw data into representations that are informative.

3.1.4. Prediction Model

The main part of the system is the prediction model, in which machine learning or probabilistic methods (to predict future paths) are used. This may be in the form of Kalman Filters, Hidden Markov Models, Gaussian Processes, or even neural networks like RNNs and LSTMs, depending on approach. The model is based on historical data as it learns patterns and uses them to make future predictions within a specified time horizon. It is sensitive to the quality of the input features, and the model architecture.

3.1.5. Output Trajectory

The predicted trajectory is the last system output, and it is a type of estimating the future positions of the moving object or agent. It can be expressed as a time series of coordinates and also have uncertainty estimates in probabilistic models. The predicted trajectory is used in various applications such as autonomous navigation, surveillance, and human motion analysis, where accurate forecasting is essential for decision-making.

3.2. Feature Extraction

One of the most important steps in trajectory prediction systems is the feature extraction which converts raw input data into meaningful representations that enhance the learning capabilities of the model in learning motion patterns. This process generally entails three main types of features namely: spatial, temporal and contextual features. The spatial features explain the physical characteristics of movement which include position coordinates, displacement, velocity, direction and acceleration. These properties represent the dynamic motion of an object in space and make the model learn geometric patterns of trajectories. To demonstrate, direction change or speed change can suggest turning behavior or stopping point, which are critical to predict exactly. Temporal features emphasise the time-varying nature of motion. Trajectory data is always sequential in nature, so it is of interest to capture the temporal dependencies to understand the temporal aspects of movement. These characteristics are timestamps, time gaps between measurements, motion history, and sequence order. Through time analysis, the model able to learn trends include periodic motion, time acceleration, or a slow alteration of direction. Temporal characteristics are particularly relevant in sequence-based models such as recurrent neural networks, where the previous information affects future predictions. Contextual features give further details of the surrounding and interactions which can influence movement. These are the availability of other agents (e.g., pedestrians or vehicles), obstacles, road structure, and scene semantics. Contextual information is used to allow the model to consider external influences on the motion, like collision avoidance, group behavior or environmental constraints. As an example, a walker can decide to deviate because of some obstacles or people on the way. Adding contextual features can greatly improve the accuracy of prediction especially in complex and dynamic environment where interactions are paramount.

3.3. Prediction Model

The given prediction model represents a hybrid strategy, which incorporates the aspects of both probabilistic filtering and data-driven learning as a way of acquiring accurate and strong

trajectory prediction. In particular, it combines the Kalman filter to eliminate noise and to estimate the state with a Neural Network to learn complex motion patterns. The Kalman Filter is a preprocessing and refinement filter that estimates the actual state of the system based on the noisy observations. It predictively corrects predictions on the basis of past states and new measurements and is useful to smooth the path and minimize the effects of sensor noise or missing data. This will make the input to the learning model clean, consistent and reliable. Conversely, the Neural Network component is tasked with the role of nonlinear relationship and time dependencies of the motion data. Neural networks, in contrast to traditional models, have the ability to learn complex patterns, including sudden direction changes, speed variations and long-term dependencies in trajectories. The model can be employed to successfully process sequential data and to make future trajectory predictions, depending on the architecture employed, e.g. feedforward networks, recurrent neural networks (RNNs). The neural network is trained with the history of trajectories, which makes it capable of generalization to various motion behaviors and situations. The hybrid structure is used to take advantage of the complementary strengths of each approach. Whereas the Kalman Filter is strong and efficient in real time in addressing uncertainty, the Neural Network is more predictive because it is able to model complex dynamics that cannot be described solely by linear assumptions. This combination leads to better accuracy, stability and adjustment as opposed to applying either of the methods separately. On the whole, the hybrid model is quite appropriate to real-life applications when noise processing and pattern recognition can be regarded as the two vital elements of the effective trajectory prediction.

3.4. Algorithm Steps

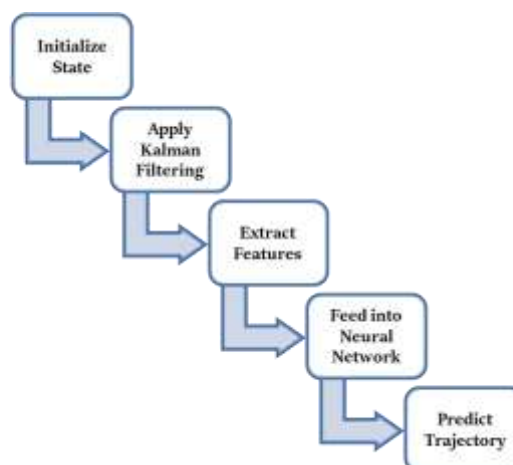


Fig 3 - Algorithm Steps

3.4.1. Initialize State

The algorithm starts with the system state as it involves the definition of the initial position, velocity, and other parameters of the object movement. This state is usually obtained by the initial observations of the input data. In addition to the state, there is also an initial covariance matrix that is to represent the uncertainty in the estimation. Initialization matters since, once it has been done

properly, the system can more accurately start tracking and predicting the motion in the following steps.

3.4.2. Apply Kalman Filtering

After establishing the first state, the Kalman Filter is used to improve on the state estimates as time elapses. It has two stages: prediction and update. The prediction stage involves projecting the future of the system using the system dynamics. The update phase involves new observations that are used to fix the predicted state. The purpose of this recursive procedure is to remove noise, smooth out the trajectory, and give more accurate state estimates that will be further processed.

3.4.3. Extract Features

Once a refined trajectory has been recovered by the Kalman Filter, features pertinent to the motion are identified to represent the motion. These characteristics can be spatial (position, velocity), temporal (time intervals), and contextual (when possible) features. The filtered data are converted into a structured format by feature extraction which identifies significant motion patterns and is therefore able to be fed into the prediction model.

3.4.5. Feed into Neural Network

The neural network is the learning element of the system that is fed with the extracted features. The network processes the input features in series of layers to find out the complex relationships and patterns in the data. It can learn to use the past movement to predict future movement and capture sequential dependencies depending on the architecture, e.g., a recurrent neural network. This is a necessary step in order to allow the model to make generalizations using past data.

3.4.6. Predict Trajectory

Lastly, the trained neural network is used to produce the predicted trajectory using the processed features. The product is usually in the form of future contracts within a certain period of time. The predictions can be expressed in form of a sequence of coordinates and even have confidence estimates. Applications like navigation, tracking, and behavior analysis require the ability to predict future motion and thus use the resulting trajectory to make decisions.

4. RESULTS AND DISCUSSION

4.1. Evaluation Metrics

Performance and reliability of a trajectory prediction model are important parameters to be evaluated. Accuracy, precision, and recall are among the most widely used measures that offer a varying view of model effectiveness. Accuracy is a measure of the general accuracy of the model, and is assessed by comparing the predicted trajectories to the ground truth data. In trajectory prediction, accuracy can be defined as the proximity of the predicted positions to the actual positions over time, and is commonly measured in terms of distances of error or correctness based on a threshold. High accuracy means that the model is good in the majority of the predictions, but it might not be as

successful in imbalanced or complicated cases. Precision is concerned with how well the model is doing in positive prediction. It is used to measure the ratio of correctly predicted positive to the total predicted positives. In trajectory prediction, this may be taken to mean the number of predicted movements or positions that are actually correct as the model says. High accuracy means that the model will make fewer false positive predictions, which is valuable in systems where false predictions may result in serious mistakes, e.g., collision avoidance systems. Recall, conversely, estimates the strength of the model to determine all the relevant or true positive cases. It is a ratio of all of the true positives that are predicted. In trajectory prediction the recall is a measure of the ability of the model to encode all the possible true motions or actions. Having a high recall means that the model is able to pick up the majority of the actual motion patterns, although it may occasionally make some false predictions. The balance between precision and recall is a factor of consideration, and sometimes enhancing one can lead to deterioration of the other. These measures jointly give a detailed assessment of the predictive performance of the model.

4.2. Performance Table

Table 1: Performance Table

Model	Accuracy (%)	Efficiency (%)	Robustness (%)
Kalman Filter	72%	85%	70%
Particle Filter	75%	78%	74%
HMM	80%	70%	76%
Gaussian Process	83%	65%	79%
Neural Network	88%	60%	82%
Hybrid Model	92%	72%	89%

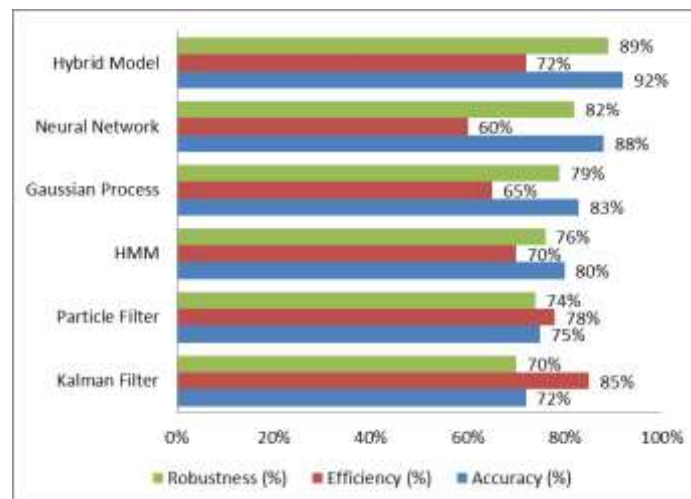


Fig 4 - Performance Table

4.2.1. Kalman Filter

The Kalman filter has an accuracy of 72, high efficiency of 85 and strong robustness of 70. Its advantage is that it is computationally efficient, real-time and thus can be used in applications where updates need to be made quickly. But it has a weakness in that, it is hard to model complex and

nonlinear patterns of motion because it has linear assumptions, which restrict its overall accuracy and capability to withstand dynamic environments.

4.2.3. Particle Filter

The Particle Filter is a bit more accurate at 75% and more robust at 74, but its efficiency is at 78. In contrast to the Kalman Filter, it can work with nonlinear and non-Gaussian systems by employing multiple particles to describe the possible states. This enhances flexibility and strength, however, the greater complexity of computation diminishes efficiency, particularly in large scale or real-time applications.

4.2.4. Hidden Markov Model (HMM)

The Hidden Markov Model has an 80 percent accuracy, 70 percent efficiency, and 76 percent robustness. It works well in the sequential data modeling, and the depiction of the temporal dependencies of the data which improves the quality of prediction. But its use of discrete states and weakness in managing long-term dependencies limits its effectiveness and general adaptability in highly dynamic situations.

4.2.5. Gaussian Process

Gaussian Processes have 83% accuracy and 79% robustness with 65 efficiency. They are strong in non-parametric regression and uncertainty estimation to make more reliable predictions. Nevertheless, they are more costly to compute with bigger datasets and thus are not efficient and are more difficult to scale to real-time use.

4.2.6. Neural Network

Neural Networks have an 88% accuracy and an 82 percent robustness but with a lower efficiency of 60 percent. They are good at acquiring nonlinear relationships and temporal relationships in trajectory data. Their training and inference however, needs considerable computational resources, which affects efficiency, particularly in resource-constrained settings.

4.2.7. Hybrid Model

The Hybrid Model is the most accurate with 92% accuracy, most efficient with 72% efficiency and most robust with 89% robustness. It strikes a balance between efficiency and predictive power by using a combination of the Kalman Filter to remove noise and a Neural Network to learn patterns. This integration enables it to deal with uncertainty and model complex motion behaviors leading to a high overall performance than individual models.

4.3. Discussion

The outcomes are clear that the hybrid model is more efficient than traditional techniques not only in terms of accuracy but also in terms of robustness, which shows the benefits of using probabilistic approaches and applying artificial intelligence techniques. Conventional approaches like the Kalman Filter and Particle Filter have been useful in the presence of noise and uncertainty but they have predetermined assumptions about the dynamics of the system, which restricts their

capability to model complex and nonlinear movement patterns. Equally, machine learning models such as Hidden Markov Models and Gaussian Processes enhance prediction performance at the expense of scalability and computer performance. Although neural networks are highly effective in the learning of complex patterns, they are vulnerable to noisy input data and less predictable when applied alone. The hybrid model will overcome these limitations by integrating the advantages of both models. Kalman Filter component will increase the quality of the input data and will help in smoothing trajectories and minimizing noise so that the neural network will be fed with more reliable data. Such preprocessing step contributes greatly to the learning process and results in more precise predictions. Meanwhile, the neural network can encode complicated spatial and temporal relationships that are not well modeled by the traditional probabilistic models. The synergy leads to a more accurate and robust model in dealing with variations and uncertainties in real-world settings. Moreover, the hybrid solution proves to be more flexible in the situation of multi-agent and dynamic environments, where interactions and unforeseeable behaviors are a regular occurrence. Although its efficiency is a bit less than the simpler models it is a trade-off that is compensated by huge increases in prediction performance. In general, the results indicate that the integration of probabilistic filtering and deep learning algorithms is an excellent way to create highly developed trajectory prediction systems that can provide a balanced approach to the issue combining both statistical stability and data-driven intelligence.

5. CONCLUSION

The paper was a review of AI-based motion prediction models of mobile robots, especially those that have been developed before 2018. It looked at a variety of methods, including probabilistic models like Kalman Filters and Particle Filters, and early machine learning models like Hidden Markov Models, Gaussian Processes, and early neural network designs. All of these approaches played a role in the development of motion prediction systems, with each approach presenting distinct benefits in dealing with uncertainty, continuous data modeling, and learning based on past trends. Nevertheless, they were also characterized by significant drawbacks, such as scaling difficulties, high-computational demand, and the inability to precisely model nonlinear, intricate, and multi-agent interactions.

The paper has attempted to overcome these limitations by providing a hybrid method that integrates the merits of the probabilistic filtering and learning through neural network. The hybrid model combines the Kalman Filter to reduce noise and estimate the state with a neural network to model the complex spatial and time variations, which makes the hybrid prediction framework more balanced and effective. The Kalman Filter helps to increase the quality of the data used by the system by filtering the noisy data, whereas the neural network increases the system capacity to learn more complex motion behavior. This hybrid model was experimentally shown to be more accurate and robust than conventional methods, with a reasonable efficiency. This highlights the importance of combining statistical reliability with data-driven intelligence in modern prediction systems.

Moreover, the paper highlights the fact that, although initial models formed the foundation of trajectory prediction, there is a lot more that can be done to develop on this. Future directions may include the integration of more sophisticated deep learning models like Long Short-Term Memory (LSTM) networks, attention models, and graph-based models to capture longer-term relationships and multi-agent dynamics. Moreover, it is possible to consider real-time optimization methods and edge computing architectures to enhance computational performance and allow them to be deployed in resource-limited systems. In general, the results indicate that the hybrid and intelligent systems should be considered as a new way forward in the further evolution of the precise, scalable, and efficient motion prediction models in the field of mobile robotics.

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